New results in time-dependent open shop scheduling

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Introduction: Classical scheduling theory vs. modern scheduling theory

- As opposed to the classical scheduling theory, where job processing times are numbers, in the modern scheduling theory job processing times are functions
- Hence, in scheduling problems considered in modern scheduling theory the parameters which describe the set of jobs or machines are variable and change in time
- One of such parameters is the processing time of a job
- In modern scheduling theory are considered variable job processing times described by
  - functions of the amounts of resources allocated to the jobs,
  - the positions of the jobs in a schedule or
  - the starting times of the jobs

Introduction: Scheduling deteriorating jobs

- Job processing times which are functions of the job starting times are called time-dependent job processing times
- Scheduling problems with time-dependent processing times are studied in view of many applications; see, e.g., Agnetis et al., 2014; Gawiejnowicz, 2020; Strusevich and Rustogi, 2017
- There are know different forms of the functions describing the time-dependent job processing times
- The most common are time-dependent processing times defined by non-decreasing functions of the job starting times
- Jobs with the latter form of time-dependent processing times are called deteriorating jobs

Problem $Om|p_{ij} = b_{ij}t|C_{\max}$

- In this talk, we consider time-dependent open shop scheduling problem with proportionally deteriorating jobs
- This problem can be formulated as follows
- We are given m dedicated machines $M_1, M_2, \ldots, M_m$ which are available for processing from time $t_0 > 0$
- We also are given n independent jobs $J_1, J_2, \ldots, J_n$, where $n \geqslant m$, have to be scheduled on machines $M_1, M_2, \ldots, M_m$
- Job J_k is composed of m operations, $O_{1k}, O_{2k}, \ldots, O_{mk}$, $1 \leqslant k \leqslant n$, and it is completed if all the operations have been completed

Problem $Om|p_{ij} = b_{ij}t|C_{\max}$

- The processing time of the ith operation of the jth job, O_{ij}, proportionally deteriorates in time and equals $p_{ij} = b_{ij}s_{ij}$, where integer *deterioration rates* $b_{ij} > 0$ for $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$, and $s_{ij} \geqslant t_0$ is the starting time of the operation
- Any job schedule satisfying the assumptions is feasible
- The criterion of schedule optimality is the makespan

$$C_{\max} = \max\{C_{ij} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\},$$

where C_{ij} is the completion time of operation O_{ij}

Problems $O2|p_{ij} = b_{ij}t|C_{\max}$ and $O3|p_{ij} = b_{ij}t|C_{\max}$

- In this talk, we focus on two special cases of problem $Om|p_{ij} = b_{ij}t|C_{\max}$ for $m = 2$ and $m = 3$
- The first and the second case will be denoted as $O2|p_{ij} = b_{ij}t|C_{\max}$ and $O3|p_{ij} = b_{ij}t|C_{\max}$, respectively

Previous research results: Open shop scheduling with fixed operation processing times

- Since the literature on open shop scheduling is quite large, we limit our brief review only to two- and three-machine open shop problems with the $C_{\max}$ criterion
- Open shop scheduling problems with the $C_{\max}$ criterion were introduced by Gonzalez and Sahni (1976)
- Gonzalez and Sahni (1976) assumed that operation processing times are fixed and proved that problem $O2||C_{\max}$ is solvable in $O(n)$ time
- The authors also proved that problem $Om||C_{\max}$ is $\mathcal{NP}$-hard for every fixed $m \geqslant 3$
- It is not known whether this problem is strongly $\mathcal{NP}$-hard or not (Woeginger, 2018)
- Sevastyanov (2019) answered in the negative a similar question when $p_{ij} = p_j$

Previous research results: Open shop scheduling with fixed operation processing times

- Research on open shop scheduling with fixed operation processing times are discussed in
  - chapter by Gonzalez (2004),
  - paper by Gawiejnowicz and Kolińska (2021)
  - review by Woeginger (2018),
  - monographs by Pinedo (1995) and Tanaev et al. (1994).

Previous research results: Open shop scheduling with time-dependent operation processing times

- Proportionally deteriorating operation processing times were introduced by Mosheiov (1994)
- The research on open shop scheduling with proportionally deteriorating processing times, compared to that one with fixed operation processing times, is very limited
- Kononov (1996) and Mosheiov (2002) independently proved that problem $O2|p_{ij} = b_{ij}t|C_{\max}$ is solvable in $O(n)$ time by a modification of the Gonzalez-Sahni algorithm
- Kononov (1996) also proved that problem $O3|p_{ij} = b_{ij}t|C_{\max}$ and problem $O3|p_{ij} = b_{ij}t, b_{3j} = b|C_{\max}$ both are $\mathcal{NP}$-hard
- Kononov and Gawiejnowicz (2001) proved that if we change proportional processing times into linear ones, then already problem $O2|p_{ij} = a_{ij} + b_{ij}t|C_{\max}$ is $\mathcal{NP}$-hard

Previous research results: Open shop scheduling with time-dependent operation processing times

- Research on time-dependent open shop scheduling problems are discussed in
  - reviews by Cheng et al. (2004) and Gawiejnowicz (2020),
  - monograph by Gawiejnowicz (2020).

A new lower bound for $Om|p_{ij} = b_{ij}t|C_{\max}$

- Our first result is a new lower bound on the value of $C_{\max}$ for problem $Om|p_{ij} = b_{ij}t|C_{\max}$
- The bound generalizes the lower bound for two-machine time-dependent open shop problem by Mosheiov (2002):

$$C^{\star}_{\max}(\sigma) \geqslant \max\left\{\prod_{j=1}^{n}(1+b_{1j}), \prod_{j=1}^{n}(1+b_{2j}), \max_{1\leqslant j\leqslant n}\{(1+b_{1j})(1+b_{2j})\}\right\}, \quad (1)$$

where σ is a feasible schedule for the problem

Theorem (Gawiejnowicz and Kolińska, 2021)

The minimum makespan of any feasible schedule σ for problem $Om|p_{ij} = b_{ij}t|C_{\max}$ satisfies the inequality

$$C^{\star}_{\max}(\sigma) \geqslant \max\left\{\max_{1\leqslant i\leqslant m}\left\{\prod_{j=1}^{n}(1+b_{ij})\right\}, \max_{1\leqslant j\leqslant n}\left\{\prod_{i=1}^{m}(1+b_{ij})\right\}\right\}. \quad (2)$$

- The proof of the result is similar to the proof of a similar lower bound for the multi-machine open shop problem with fixed processing times

The LADR rule for $O2|p_{ij} = b_{ij}t|C_{\max}$: Formulation

- Our second result concerns problem $O2|p_{ij} = b_{ij}t|C_{\max}$
- We propose to solve this problem using the following rule

  whenever a machine becomes free, assign to the machine this job was not yet processed on either machine and which has the largest deterioration rate on the other machine

- We will call the new rule the *Largest Alternate Deterioration Rate first (LADR)* rule
- The LADR rule is a time-dependent counterpart of the LAPT rule by Pinedo (1995)
- The LADR rule assigns a job to the freed machine taking into account the deterioration rate of the job on the other machine

The LADR rule for $O2|p_{ij} = b_{ij}t|C_{\max}$: The main result

- If both machines are idle and deterioration rates of the same job on both machines are equal, this job may be assigned to any of both machines
- Jobs that already have been completed on the other machine, will be assign to the machine just freed with the lowest priority

Theorem (Gawiejnowicz and Kolińska, 2021)

Problem $O2|p_{ij} = b_{ij}t|C_{\max}$ is solvable by the LADR rule and the makespan of schedule constructed by the rule satisfies bound (1).

- This result may be proved either by adopting the original proof by Pinedo (1995) or by applying the notion of isomorphic scheduling problems (Gawiejnowicz and Kononov, 2014)

The LADR rule for $O2|p_{ij} = b_{ij}t|C_{max}$: Example

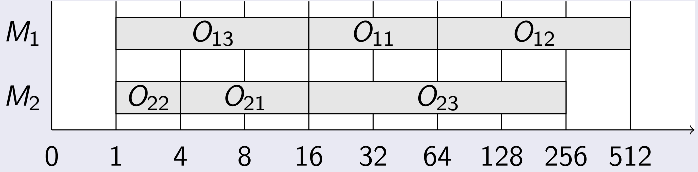
Example (Gawiejnowicz and Kolińska, 2021)

- Let us consider an instance of problem $O2|p_{ij} = b_{ij}t|C_{max}$ with $t_0 = 1$ and $n = 3$ jobs with deterioration rates as below

j	b_{1j}	b_{2j}
1	3	7
2	7	1
3	15	15

Example (Gawiejnowicz and Kolińska, 2021)

- The LADR rule generates for this instance the schedule presented below
- Since the lower bound (1) for the instance equals 512 time units and since the makespan for this schedule equals 512 time units as well, the schedule is optimal.



The LADR rule for $O2|p_{ij} = b_{ij}t|C_{\max}$: Isomorphic schedules

Isomorphic schedules for problems $O2||C_{\max}$ and $O2|p_{ij} = b_{ij}t|C_{\max}$

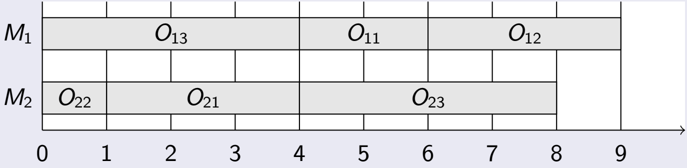
- The previous instance of problem $O2|p_{ij} = b_{ij}t|C_{\max}$ corresponds to an instance of problem $O2||C_{\max}$, defined as below

j	p_{1j}	p_{2j}
1	2	3
2	3	1
3	4	4

The LADR rule for $O2|p_{ij} = b_{ij}t|C_{\max}$: Isomorphic schedules

Isomorphic schedules for problems $O2||C_{\max}$ and $O2|p_{ij} = b_{ij}t|C_{\max}$

- The LAPT rule generates for this instance the schedule presented below



- Since the lower bound for the instance equals $\max\{9, 8, 8\} = 9$ time units, the schedule is optimal
- The structures of presented two schedules are identical, differences concern the starting and completion times of operations
- This is caused by the fact that problems $O2||C_{\max}$ and $O2|p_{ij} = b_{ij}t|C_{\max}$ are isomorphic (Gawiejnowicz and Kononov, 2014)
- Hence, these schedules may be called isomorphic schedules

The LTRDROM rule for $O3|p_{ij} = b_{ij}t|C_{\max}$: Formulation

- Our last result concerns problem $O3|p_{ij} = b_{ij}t|C_{\max}$
- We propose to solve this problem using a new scheduling rule:

  every time a machine is freed, the job with the largest total remaining deterioration rate on all other machines, among available jobs, is selected for processing.
- We will call the rule the *Largest Total Remaining Deterioration Rate on Other Machines first* (*LTRDROM*)
- The quality of schedules generated by the LTRDROM rule was tested in numerical experiments
- In total, we tested 240 instances: 120 small-size instances with $5 \leqslant n \leqslant 15$, and 120 medium-size instances with $20 \leqslant n \leqslant 30$
- The average computation time varied between 1.1191 ms for instances with $n = 5$ jobs and 7.4223 ms for instances with $n = 30$ jobs

The LTRDROM rule for $O3|p_{ij} = b_{ij}t|C_{max}$: Results of numerical experiments

- From 120 tested small-size instances, the LTRDROM rule generated optimal schedules for more than 55% of cases

n	$Range$	$Time$	ρ_R^{avg}	$\#OPT$	C_{max}^{avg}
5	1-5	1.1191	1.1773	7	1,907.8
5	5-10	1.2110	4.3726	5	320,385.2
5	10-20	1.4654	10.8350	4	21,814,889.7
5	20-30	1.3756	80.1805	3	1,200,738,748.4
10	1-5	1.8010	1.1880	9	2,348,136.0
10	5-10	1.5888	1.8137	5	4,681,800,340.8
10	10-20	2.5963	5.6171	3	12,132,906,667,660.8
10	20-30	1.7471	29.0355	6	5,010,289,585,173,965.0
15	1-5	2.3739	1.0919	8	1,456,132,032.0
15	5-10	2.9033	2.1920	6	237,040,565,241,715.2
15	10-20	2.6479	3.4653	4	5,785,592,054,664,587,300.0
15	20-30	3.5196	1.1019	7	15,123,215,284,498,608,000.0

The LTRDROM rule for $O3|p_{ij} = b_{ij}t|C_{max}$: Results of numerical experiments

- A better percentage was observed for 120 medium-size instances with 20, 25 or 30 jobs, where more than 63% of instances were solved optimally

n	*Range*	*Time*	ρ_R^{avg}	$\#OPT$	C_{max}^{avg}
20	1-5	4.0947	1.0000	10	2,346,572,851,200.0
20	5-10	5.7421	1.6923	5	5,747,203,919,569,305,600.0
20	10-20	5.5698	1.2507	5	16,165,384,563,975,053,000.0
20	20-30	4.2232	1.0614	7	15,189,746,438,801,412,000.0
25	1-5	5.2487	1.0481	9	4,008,844,343,232,000.0
25	5-10	5.5889	1.0265	7	16,260,667,245,228,392,000.0
25	10-20	6.5342	1.5490	3	16,182,875,126,184,071,000.0
25	20-30	8.3018	1.5398	4	16,973,710,572,573,786,000.0
30	1-5	6.3513	1.0000	10	3,348,713,572,761,600,000.0
30	5-10	8.4471	1.1788	6	13,885,565,913,806,078,000.0
30	10-20	7.5279	1.1559	6	15,232,833,398,617,758,000.0
30	20-30	7.4223	1.4188	4	14,820,357,698,650,059,000.0

Conclusions

- We considered two- and three-machine open shop problems with proportionally deteriorating jobs and the $C_{\max}$ criterion
- For the first problem, we proposed a new scheduling rule, LADR, and proved that it is optimal for the problem
- For the second problem, we proposed another new scheduling rule, LTRDROM, and tested it in numerical experiments with small- and medium-size instances
- Results of the experiments suggest that the LTRDROM rule generates near-optimal schedules for small- and medium-size instances of problem $O3|p_{ij} = b_{ij}t|C_{\max}$
- Future research on problem $O3|p_{ij} = b_{ij}t|C_{\max}$ may concern the construction of branch-and-bound algorithms or heuritics combining a good initial solution with lower bound (2)

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Thank you
for your attention!